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APPLICATION OF MAHGOUB TRANSFORM FOR SOLVING LINEAR VOLTERRA INTEGRAL EQUATIONS OF FIRST KIND

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ABSTRACT

Many advance problems of biology, chemistry, physics and engineering can represent mathematically in the form of Volterra integral equations of first kind. In this paper, we used Mahgoub transform for solving linear Volterra integral equations of first kind and some applications are given in order to demonstrate the effectiveness of Mahgoub transform for solving linear Volterra integral equations of first kind.

Keywords: *Linear Volterra integral equation of first kind, Mahgoub transform, Convolution theorem, Inverse Mahgoub transform*

I. INTRODUCTION

The linear Volterra integral equation of first kind is given by [1-12]

$$f(x) = \int_0^x k(x,t)u(t)dt \text{ .... } (1)$$

where the unknown function $u(x)$, that will be determined, occurs only inside the integral sign. The kernel $k(x,t)$and the function $f(x)$ are given real-valued functions.

The Mahgoub transform of the function $F(t)$is defined as [16, 24]:

$$M\{F(t)\} = v\int_0^\infty F(t)e^{-vt}dt = H(v), t \geq 0, k_1 \leq v \leq k_2$$

where M is Mahgoub transform operator.

The Mahgoub transform of the function $F(t)$ for $t \geq 0$ exist if $F(t)$ is piecewise continuous and of exponential order. These conditions are only sufficient conditions for the existence of Mahgoub transform of the function $F(t)$. Mahgoub and Alshikh [17] used Mahgoub transform for solving partial differential equations. Fadhil [18] discussed the convolution for Kamal and Mahgoub transforms. Taha et. al. [19] gave the dualities between Kamal & Mahgoub integral transforms and some famous integral transforms. Aggarwal et al. [20] discussed the new application of Mahgoub transform for solving linear ordinary differential equations with variable coefficients. A new application of Mahgoub transform for solving linear Volterra integral equations was given by Aggarwal et al. [21]. Aggarwal et al. [22] solved linear Volterra integro-differential equations of second kind using Mahgoub transform. Aggarwal et al. [23] gave Mahgoub transform of Bessel's functions. Kiwne and Sonawane [24] discussed Mahgoub transform fundamental properties and applications.

The aim of this work is to establish exact solutions for linear Volterra integral equation of first kind using Mahgoub transform without large computational work.

II. LINEARITY PROPERTY OF MAHGOUB TRANSFORM [23]

If $M\{F(t)\} = H(v)$ and $M\{G(t)\} = I(v)$ then $M\{aF(t) + bG(t)\} = aM\{F(t)\} + bM\{G(t)\}$
$\Rightarrow M\{aF(t) + bG(t)\} = aH(v) + bI(v)$,
where a, b are arbitrary constants.

III. MAHGOUB TRANSFORM OF SOME ELEMENTARY FUNCTIONS [24]

S.N.	$F(t)$	$M\{F(t)\} = H(v)$
1.	1	1
2.	t	$\frac{1}{v}$
3.	t^2	$\frac{2!}{v^2}$
4.	$t^n, n \in N$	$\frac{n!}{v^n}$
5.	$t^n, n > -1$	$\frac{\Gamma(n+1)}{v^n}$
6.	e^{at}	$\frac{v}{v-a}$
7.	$sinat$	$\frac{av}{v^2+a^2}$
8.	$cosat$	$\frac{v^2}{v^2+a^2}$
9.	$sinhat$	$\frac{av}{v^2-a^2}$
10.	$coshat$	$\frac{v^2}{v^2-a^2}$

IV. CONVOLUTION OF TWO FUNCTIONS [15]

Convolution of two functions $F(t)$ and $G(t)$ is denoted by $F(t) * G(t)$ and it is defined by

$$F(t) * G(t) = F * G = \int_0^t F(x)G(t-x)dx = \int_0^t F(t-x)G(x)dx$$

V. CONVOLUTION THEOREM FOR MAHGOUB TRANSFORMS [18, 21-24]

If $M\{F(t)\} = H(v)$ and $M\{G(t)\} = I(v)$ then $M\{F(t) * G(t)\} = \frac{1}{v}M\{F(t)\}M\{G(t)\} = \frac{1}{v}H(v)I(v)$

VI. INVERSE MAHGOUB TRANSFORM [21-23]

If $M\{F(t)\} = H(v)$ then $F(t)$ is called the inverse Mahgoub transform of $H(v)$ and mathematically it is defined as
$F(t) = M^{-1}\{H(v)\}$

where M^{-1} is the inverse Mahgoub transform operator.

VII. INVERSE MAHGOUB TRANSFORM OF SOME ELEMENTARY FUNCTIONS [21-23]

S.N.	$H(v)$	$F(t)=M^{-1}\{H(v)\}$
1.	1	1
2.	$\frac{1}{v}$	t
3.	$\frac{1}{v^2}$	$\frac{t^2}{2!}$
4.	$\frac{1}{v^n}, n\in N$	$\frac{t^n}{n!}$
5.	$\frac{1}{v^n}, n>-1$	$\frac{t^n}{\Gamma(n+1)}$
6.	$\frac{v}{v-a}$	e^{at}
7.	$\frac{v}{v^2+a^2}$	$\frac{sinat}{a}$
8.	$\frac{v^2}{v^2+a^2}$	$cosat$
9.	$\frac{v}{v^2-a^2}$	$\frac{sinhat}{a}$
10.	$\frac{v^2}{v^2-a^2}$	$coshat$

VIII. MAHGOUB TRANSFORM OF BESSEL'S FUNCTIONS [23]

a) Mahgoub transform of Bessel's function of zero order $J_0(t)$:

$$M\{J_0(t)\}=\frac{v}{\sqrt{(1+v^2)}}$$

b) Mahgoub transform of Bessel's function of order one $J_1(t)$:

$$M\{J_1(t)\}=v-\frac{v^2}{\sqrt{(1+v^2)}}$$

IX. MAHGOUB TRANSFORMS FOR LINEAR VOLTERRA INTEGRAL EQUATIONS OF FIRST KIND

In this work we will assume that the kernel $k(x,t)$of (1) is a difference kernel that can be expressed by the difference$(x-t)$. The linear Volterra integral equatin of first kind (1) can thus be expressed as

$$f(x)=\int_0^x k(x-t)u(t)dt \;\ldots\ldots\ldots\ldots\ldots (2)$$

Applying the Mahgoub transform to both sides of(2), we have

$$M\{f(x)\}=M\{\int_0^x k(x-t)u(t)dt\}\ldots\ldots (3)$$

Using convolution theorem of Mahgoub transform, we have

$$M\{f(x)\}=\frac{1}{v}M\{k(x)\}M\{u(x)\}$$

$$\Rightarrow M\{u(x)\}=v\left[\frac{M\{f(x)\}}{M\{k(x)\}}\right]\ldots\ldots\ldots\ldots (4)$$

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Operating inverse Mahgoub transform on both sides of(4), we have

$$u(x) = M^{-1}\left\{v\left[\frac{M\{f(x)\}}{M\{k(x)\}}\right]\right\} \ldots\ldots\ldots.(5)$$

which is the required solution of (2).

X. APPLICATIONS

In this section, some applications are given in order to demonstrate the effectiveness of Mahgoub transform for solving linear Volterra integral equations of first kind.

A. Application:1 Consider linear Volterra integral equation of first kind

$$x = \int_0^x e^{(x-t)}\, u(t)dt \ldots\ldots\ldots\ldots (6)$$

Applying the Mahgoub transform to both sides of(6), we have

$$M\{x\} = M\{\int_0^x e^{(x-t)}\, u(t)dt\} \ldots\ldots\ldots\ldots (7)$$

Using convolution theorem of Mahgoub transform on (7), we have

$$\frac{1}{v} = \frac{1}{v} M\{e^x\} M\{u(x)\}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{v}\left[\frac{v}{v-1}\right] M\{u(x)\}$$

$$\Rightarrow M\{u(x)\} = \frac{v-1}{v} \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Operating inverse Mahgoub transform on both sides of(8), we have

$$u(x) = M^{-1}\left\{\frac{v-1}{v}\right\} = M^{-1}\{1\} - M^{-1}\left\{\frac{1}{v}\right\}$$

$$\Rightarrow u(x) = 1 - x \ldots\ldots\ldots\ldots (9)$$

which is the required exact solution of (6).

B. Application:2 Consider linear Volterra integral equation of first kind

$$sinx = \int_0^x e^{(x-t)} u(t)\, dt \ldots\ldots (10)$$

Applying the Mahgoub transform to both sides of(10), we have

$$M\{sinx\} = M\{\int_0^x e^{(x-t)} u(t)\, dt\} \ldots\ldots\ldots\ldots (11)$$

Using convolution theorem of Mahgoub transform on(11), we have

$$\frac{v}{1+v^2} = \frac{1}{v} M\{e^x\} M\{u(x)\}$$

$$\Rightarrow \frac{v}{1+v^2} = \frac{1}{v}\left[\frac{v}{v-1}\right] M\{u(x)\}$$

$$\Rightarrow M\{u(x)\} = \frac{v(v-1)}{1+v^2} = \frac{v^2}{1+v^2} - \frac{v}{1+v^2} \ldots\ldots (12)$$

Operating inverse Mahgoub transform on both sides of(12), we have

$$u(x) = M^{-1}\left\{\frac{v^2}{1+v^2}\right\} - M^{-1}\left\{\frac{v}{1+v^2}\right\}$$

$$\Rightarrow\ u(x) = cosx - sinx \ldots\ldots\ldots\ldots (13)$$

which is the required exact solution of (10).

C. Application:3 Consider linear Volterra integral equation of first kind

$$sinx = \int_0^x J_0(x-t) u(t)\, dt \ldots\ldots (14)$$

Applying the Mahgoub transform to both sides of(14), we have

$$M\{sinx\} = M\{\int_0^x J_0(x-t) u(t)\, dt\} \ldots. (15)$$

Using convolution theorem of Mahgoub transform on(15), we have

$$\frac{v}{1+v^2} = \frac{1}{v} M\{J_0(x)\} M\{u(x)\}$$

$$\Rightarrow \frac{v}{1+v^2} = \frac{1}{v}\left[\frac{v}{\sqrt{(1+v^2)}}\right] M\{u(x)\}$$

$$\Rightarrow M\{u(x)\} = \frac{v}{\sqrt{(1+v^2)}} \ldots\ldots\ldots\ldots..(16)$$

Operating inverse Mahgoub transform on both sides of(16), we have

$$u(x) = M^{-1}\left\{\frac{v}{\sqrt{(1+v^2)}}\right\} = J_0(x) \ldots\ldots\ldots\ldots.. (17)$$

which is the required exact solution of (14).

D. Application:4 Consider linear Volterra integral equation of first kind

$$x^2 = \frac{1}{2}\int_0^x (x-t)u(t)\,dt\ldots\ldots (18)$$

Applying the Mahgoub transform to both sides of(18), we have

$$M\{x^2\} = \frac{1}{2} M\{\int_0^x (x-t)u(t)\,dt\}\ldots. (19)$$

Using convolution theorem of Mahgoub transform on(19), we have

$$\frac{2}{v^2} = \frac{1}{2}.\frac{1}{v} M\{x\}M\{u(x)\}$$

$$\Rightarrow \frac{2}{v^2} = \frac{1}{2}.\frac{1}{v}\left[\frac{1}{v}\right] M\{u(x)\}$$

$$\Rightarrow M\{u(x)\} = 4 \ldots\ldots\ldots\ldots..(20)$$

Operating inverse Mahgoub transform on both sides of(20), we have

$$u(x) = 4M^{-1}\{1\} = 4\ldots\ldots\ldots.. (21)$$

which is the required exact solution of (18).

E. Application:5 Consider linear Volterra integral equation of first kind

$$x = \int_0^x e^{-(x-t)} u(t)dt\ldots\ldots(22)$$

Applying the Mahgoub transform to both sides of (22), we have

$$M\{x\} = M\{\int_0^x e^{-(x-t)} u(t)dt\}\ldots. (23)$$

Using convolution theorem of Mahgoub transform on(23), we have

$$\frac{1}{v} = \frac{1}{v} M\{e^{-x}\}M\{u(x)\}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{v}\left[\frac{v}{v+1}\right] M\{u(x)\}$$

$$\Rightarrow M\{u(x)\} = \frac{v+1}{v}\ldots\ldots\ldots.. (24)$$

Operating inverse Mahgoub transform on both sides of(24), we have

$$u(x) = M^{-1}\left\{\frac{v+1}{v}\right\} = M^{-1}\{1\} + M^{-1}\left\{\frac{1}{v}\right\}$$

$$\Rightarrow u(x) = 1 + x\ldots\ldots\ldots\ldots.. (25)$$

which is the required exact solution of (22).

F. Application:6 Consider linear Volterra integral equation of first kind

$$sinx = \int_0^x u(x-t)\,u(t)dt\ldots\ldots(26)$$

Applying the Mahgoub transform to both sides of (26), we have

$$M\{sinx\} = M\{\int_0^x u(x-t)\,u(t)dt\}\ldots. (27)$$

Using convolution theorem of Mahgoub transform on(27), we have

$$\frac{v}{v^2+1} = \frac{1}{v} M\{u(x)\}M\{u(x)\}$$

$$\Rightarrow [M\{u(x)\}]^2 = \frac{v^2}{v^2+1}$$

$$\Rightarrow M\{u(x)\} = \pm\frac{v}{\sqrt{v^2+1}}\ldots\ldots\ldots.. (28)$$

Operating inverse Mahgoub transform on both sides of(28), we have

$$u(x) = \pm M^{-1}\left\{\frac{v}{\sqrt{v^2+1}}\right\}$$
$$\Rightarrow u(x) = \pm J_0(x) \ldots\ldots\ldots\ldots (29)$$

which is the required exact solution of (26).

G. Application:7 Consider linear Volterra integral equation of first kind

$$x = \int_0^x u(t)\,dt \ldots\ldots(30)$$

Applying the Mahgoub transform to both sides of (30), we have

$$M\{x\} = M\left\{\int_0^x u(t)\,dt\right\} \ldots.(31)$$

Using convolution theorem of Mahgoub transform on(31), we have

$$\frac{1}{v} = \frac{1}{v}M\{1\}M\{u(x)\}$$
$$\Rightarrow \frac{1}{v} = \frac{1}{v}.1.M\{u(x)\}$$
$$\Rightarrow M\{u(x)\} = 1 \ldots\ldots\ldots\ldots (32)$$

Operating inverse Mahgoub transform on both sides of(32), we have

$$u(x) = M^{-1}\{1\} = 1 \ldots\ldots\ldots\ldots (33)$$

which is the required exact solution of (30).

H. Application:8 Consider linear Volterra integral equation of first kind

$$1 - J_0(x) = \int_0^x u(t)\,dt \ldots\ldots(34)$$

Applying the Mahgoub transform to both sides of (34), we have

$$M\{1\} - M\{J_0(x)\} = M\left\{\int_0^x u(t)\,dt\right\} \ldots.(35)$$

Using convolution theorem of Mahgoub transform on(35), we have

$$1 - \frac{v}{\sqrt{(1+v^2)}} = \frac{1}{v}M\{1\}M\{u(x)\}$$
$$\Rightarrow 1 - \frac{v}{\sqrt{(1+v^2)}} = \frac{1}{v}.1.M\{u(x)\}$$
$$\Rightarrow M\{u(x)\} = v - \frac{v^2}{\sqrt{(1+v^2)}} \ldots\ldots\ldots\ldots (36)$$

Operating inverse Mahgoub transform on both sides of(36), we have

$$u(x) = M^{-1}\left\{v - \frac{v^2}{\sqrt{(1+v^2)}}\right\} = J_1(x) \ldots\ldots\ldots\ldots (37)$$

which is the required exact solution of (34).

I. Application:9 Consider linear Volterra integral equation of first kind

$$J_0(x) - cosx = \int_0^x J_0(x-t)u(t)\,dt \ldots\ldots(38)$$

Applying the Mahgoub transform to both sides of (38), we have

$$M\{J_0(x)\} - M\{cosx\} = M\left\{\int_0^x J_0(x-t)u(t)\,dt\right\} \ldots(39)$$

Using convolution theorem of Mahgoub transform on(39), we have

$$\frac{v}{\sqrt{(1+v^2)}} - \frac{v^2}{v^2+1} = \frac{1}{v}M\{J_0(x)\}M\{u(x)\}$$
$$\Rightarrow \frac{v}{\sqrt{(1+v^2)}} - \frac{v^2}{v^2+1} = \frac{1}{v}.\frac{v}{\sqrt{(1+v^2)}}.M\{u(x)\}$$
$$\Rightarrow M\{u(x)\} = v - \frac{v^2}{\sqrt{(1+v^2)}} \ldots\ldots\ldots\ldots (40)$$

Operating inverse Mahgoub transform on both sides of(40), we have

$$u(x) = M^{-1}\left\{v - \frac{v^2}{\sqrt{(1+v^2)}}\right\} = J_1(x) \ldots\ldots\ldots\ldots (41)$$

which is the required exact solution of (38).

J. Application:10 Consider linear Volterra integral equation of first kind

$$cosx - J_0(x) = -\int_0^x J_1(x-t)u(t)\,dt \ldots(42)$$

Applying the Mahgoub transform to both sides of (42), we have

$$M\{cosx\} - M\{J_0(x)\} = -M\{\int_0^x J_1(x-t)u(t)\,dt\} \ldots. (43)$$

Using convolution theorem of Mahgoub transform on(43), we have

$$\frac{v^2}{v^2+1} - \frac{v}{\sqrt{(1+v^2)}} = -\frac{1}{v}M\{J_1(x)\}M\{u(x)\}$$

$$\Rightarrow \frac{v^2}{v^2+1} - \frac{v}{\sqrt{(1+v^2)}} = -\frac{1}{v}.\left[v - \frac{v^2}{\sqrt{(1+v^2)}}\right].M\{u(x)\}$$

$$\Rightarrow M\{u(x)\} = \frac{v}{\sqrt{(1+v^2)}} \ldots\ldots\ldots\ldots (44)$$

Operating inverse Mahgoub transform on both sides of(44), we have

$$u(x) = M^{-1}\left\{\frac{v}{\sqrt{(1+v^2)}}\right\} = J_0(x) \ldots\ldots\ldots\ldots (45)$$

which is the required exact solution of (42).

XI. CONCLUSION

In this paper, we have successfully developed the Mahgoub transform for solving linear Volterra integral equations of first kind. The given applications showed that the exact solution have been obtained using very less computational work and spending a very little time. The proposed scheme can be applied for other linear Volterra integral equations and their system.

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